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Dielectric coefficient of QCD medium from mesons

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Abstract The mesons are studied in the general framework of the hydrogenic systems. Using the known energy levels of various mesons ($Q\bar{q}$, $q\bar{q}$, $Q\bar{Q}$) of different flavour combinations, we extract the dielectric property of the colour medium in which the quarks (Q, $\bar{q}$) are present. We also estimate the sizes of these mesons as the "Bohr radius" of the quark atom in the respective state. We found that the dielectric function behaves as $\epsilon \sim a_n \langle r_n \rangle_{\rm Bohr}^{0.1}$ in different cases of the quark atomic systems and it is shown that this behaviour supports the power law potential for the confinement of coloured quarks.

Keywords Dielectric coefficient, quark-atoms, Bohm radii

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1. Introduction

Hadrons having combinations of heavy quarks with light quarks exhibit properties similar to an atom with the light quark (q) playing the role of the electron around the heavy quark (Q). The like quark-antiquark systems and baryons like Qqq then behave similar to positronium and helium like systems respectively. The mesons like $b\bar{d}$, $c\bar{u}$ actually resemble the hydrogen like quark atom with the b and c quarks as their nuclei. There are interesting heavy-light symmetry that helps one to organise the hadronic structure and properties of heavy-light systems [1]. A number of relations follow both in the spectrum and among form factors of heavy mesons when one take the mass of heavy quark (Q) to infinity [2, 3].

In this paper, we analyse the energy spectrum of these mesons in the general frame work of the hydrogen atom and extract the quantum chromo dynamics (QCD) effects through the colour dielectric constants for each of the mesons under study. We make use of the masses of these mesons from a successful confinement model for getting the colour dielectric constants. The confinement of the coloured quarks in this case is provided by the colour dielectric medium. In the next section we discuss the generalities of the quark-atoms in terms of the properties of the hydrogen atom. The relevant scaling for the coulomb energy as well as the

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'Bohr radii' for the quark-atoms are also discussed. A general form of the dielectric constants in terms of the respective 'Bohr radii' are obtained through a parameter fit and details are included in Section 2. In Section 3, we discuss the relevant link between the colour dielectric function and the phenomenological confinement potential that one adopts for the study of hadrons in terms of its quark structure. Finally in Section 4, we conclude by highlighting the relevance and scope of the present study on quark atoms.

2. Colour dielectric constants from the spectra of mesons

In this section we largely adopt the properties of hydrogen atom and generalise it for the study of various mesons ($Q\bar{q}$, $Q\bar{Q}$, $q\bar{q}$). The forces between the quark and anti-quark are best visualised with the help of Gauss's law [4]. At short distance the inter quark potential is described by an effective coulomb potential. Thus in general the mass of a $q_1\bar{q}_2$ meson can be expressed as

$$M(q_1\bar{q}_2) = M_{q_1} + M_{q_2} + E_n^{coul}(q_1\bar{q}_2 \tag{1}$$

Here $M_{q_{1,2}}$ represent masses of q_1 and $\bar{q}_2$ quarks, and $E_n^{coul}(q_1\bar{q}_2)$ is the coulomb energy that binds the two quarks. For simplicity we avoid the spin dependent part in the mass expression. The coulomb energy for a quark atom $q_1\bar{q}_2$ can be obtained as

$$E_n^{coul}(q_1\bar{q}_2) = f_{q_2}^2 f_{q_1}^2 \left(\frac{\mu_{q_1\bar{q}_2}}{\mu_H}\right) \frac{13.6}{\varepsilon_r^2(q_1\bar{q}_2)n^2}\ eV \tag{2}$$

and the "Bohr radius" of the quark atom is given in terms of the Bohr radius of the hydrogen atom as

$$\langle r_n \rangle_{q_1q_2} = \frac{\varepsilon_r(q_1\bar{q}_2)}{(\mu_{q_1\bar{q}_2}/\mu_H)f_q^2} \langle r_n \rangle_H, \tag{3}$$

where f_{q_1/q_2} are the fractional charges of the quarks in terms of the electric charge, $\mu_{q_1q_2/H}$ are the reduced mass of the light quark in the $Q\bar{q}$ system and that of the electrons in the hydrogen atom respectively as in the case of positronium ($\mu_H = m_e/2$), $\mu_{q_1\bar{q}_2}$ becomes $m_{q_1}/2$ for like flavoured mesons. $\varepsilon_r(\bar{q}Q)$ is the relative dielectric constant of the QCD medium, n is the principal energy quantum number and $\langle r_n \rangle_H$ is the Bohr radius of the hydrogen atom in the nth state. From the contour plots of the ground state masses of the like flavour mesons (quarkonium) as a function of the mass and $\varepsilon_r(q\bar{q})$, we could fix the masses of the quarks as $m_u = m_d = 160\,MeV$, $m_s = 360\,MeV$, $M_c = 1400\,MeV$ and $M_b = 4300\,MeV$. The f_q values of the c, u quarks are 2/3 and that of b, s, d quarks are – 1/3 respectively. With the help of these values and using eq. (2) in eq. (1) we get the masses of the $q_1\bar{q}_2$ mesons in terms of ε_r, the dielectric constant of the colour medium. Hence to obtain the colour dielectric property of $q_1\bar{q}_2$ systems of different quark combinations we make use of the known masses of $q_1\bar{q}_2$ mesons obtained from our earlier studies based on a relativistic harmonic confinement model [5]. With the values of ε_r thus obtained, we evaluate the "Bohr radius" of the respective quark-atom using eq. (3). Here the Bohr radius of the ground state hydrogen atom is taken as 0.529 A^o. The values of the

Table 1. Coulomb energy, the dielectric constants and the Bohr radii of different flavoured mesons :

State	Mass (MeV)	$E_{Coul}(n)$ (MeV)	$\epsilon_i \times 10^{-3}$	r_n (fm)
$b\bar{b}$ (1S)	9452	852	0.90583	0.09688
$b\bar{b}$ (2S)	10023	1423	0.35230	0.15826
$b\bar{b}$ (3S)	10326	1726	0.21326	0.21440
$b\bar{b}$ (4S)	10575	1975	0.14952	0.26866
$b\bar{c}$ (1S)	6300	600	1.52105	0.08279
$b\bar{c}$ (2S)	6951	1251	0.52670	0.12041
$b\bar{c}$ (3S)	7322	1622	0.30837	0.15778
$b\bar{c}$ (4S)	7630	1930	0.21202	0.19388
$b\bar{s}$ (1S)	5382	722	0.38882	0.26915
$b\bar{s}$ (2S)	6290	1630	0.12939	0.37618
$b\bar{s}$ (3S)	6774	2114	0.07574	0.49286
$b\bar{s}$ (4S)	7158	2498	0.05226	0.60775
$b\bar{u}$ (1S)	5222	762	0.51563	0.19216
$b\bar{u}$ (2S)	6224	1764	0.16951	0.26532
$b\bar{u}$ (3S)	6743	2283	0.09934	0.34798
$b\bar{u}$ (4S)	7149	2689	0.06865	0.42979
$c\bar{c}$ (1S)	3068	268	3.70569	0.30433
$c\bar{c}$ (2S)	3674	874	1.02601	0.35390
$c\bar{c}$ (3S)	4073	1273	0.56676	0.43753
$c\bar{c}$ (4S)	4420	1620	0.37681	0.51988
$c\bar{s}$ (1S)	2085	325	1.07615	0.86415
$c\bar{s}$ (2S)	2805	1045	0.30008	1.01203
$c\bar{s}$ (3S)	3274	1514	0.16620	1.25451
$c\bar{s}$ (4S)	3673	1913	0.11089	1.49598
$c\bar{u}$ (1S)	1912	352	1.46446	0.58631
$c\bar{u}$ (2S)	2681	1121	0.41031	0.68995
$c\bar{u}$ (3S)	3172	1612	0.22811	0.85846
$c\bar{u}$ (4S)	3584	2024	0.15268	1.02694
$s\bar{s}$ (1S)	1020	300	0.44402	0.56724
$s\bar{s}$ (2S)	1775	1055	0.11839	0.63521
$s\bar{s}$ (3S)	2292	1572	0.06466	0.77644
$s\bar{s}$ (4S)	2729	2009	0.04290	0.92063
$s\bar{u}$ (1S)	817	297	0.70015	0.36337
$s\bar{u}$ (2S)	1603	1083	0.18333	0.39960
$s\bar{u}$ (3S)	2135	1615	0.10008	0.48825
$s\bar{u}$ (4S)	2580	2040	0.06646	0.57948
$u\bar{u}$ (1S)	612	292	1.20016	0.86243
$u\bar{u}$ (2S)	1441	1121	0.30627	0.92430
$u\bar{u}$ (3S)	1987	1667	0.16743	1.13098
$u\bar{u}$ (4S)	2441	2121	0.11133	1.34398

dielectric constant and the "Bohr radius" of the various mesons are calculated up to 4S ($L = 0$) states and are tabulated in Table 1. As expected for the colour medium [6, 7] we obtain $\varepsilon_r << 1$ in all the cases, clearly indicating the anti-screening property of the QCD medium. The "Bohr radius" of these quark atoms are also found to be in the right order corresponding to the size of these mesons. The values of ε_r's against the respective "Bohr radii" for each of the quark-atoms are plotted and are shown in Figure 1.

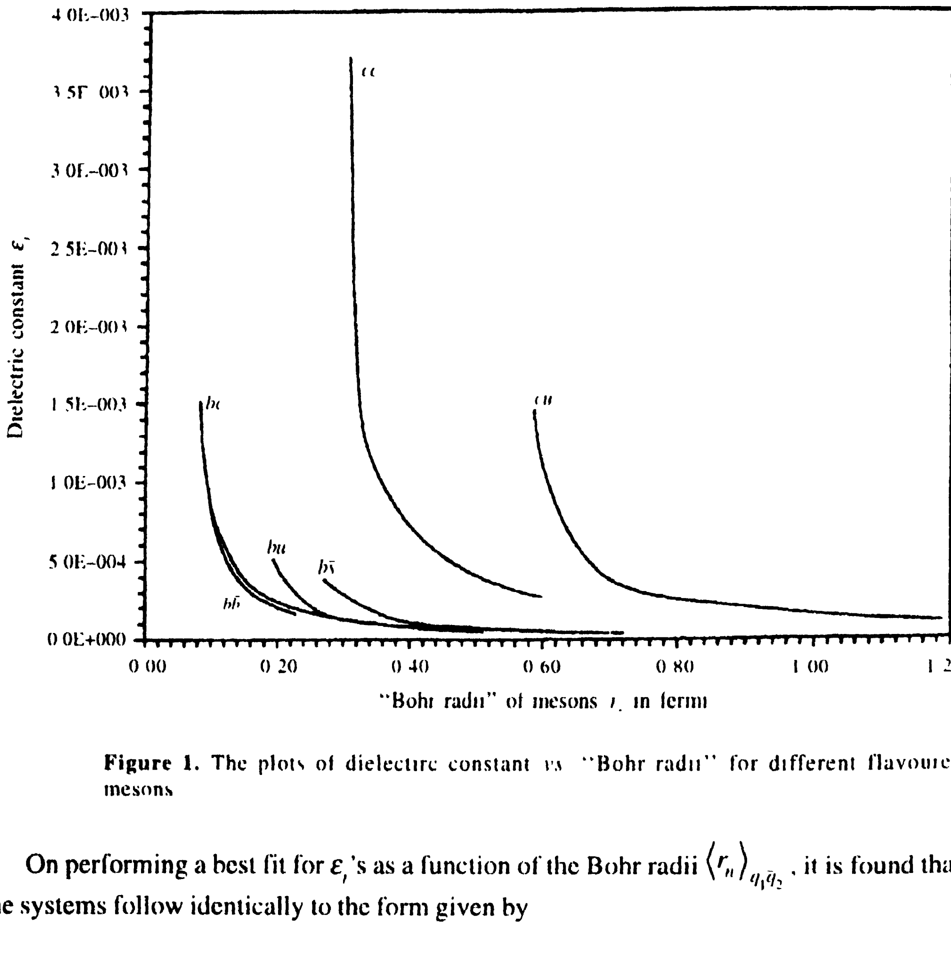

Figure 1. The plots of dielectirc constant vs "Bohr radii" for different flavoured mesons

On performing a best fit for ε_r's as a function of the Bohr radii $\langle r_n \rangle_{q_1\bar{q}_2}$, it is found that all the systems follow identically to the form given by

$$\varepsilon_r = c - a_n \langle r_n \rangle^p_{q_1\bar{q}_2} \tag{4}$$

$$\text{with } a_n = \frac{a}{E^{q_1\bar{q}_2}_{coul}(n) + m_q} \text{ and the index } p = 0.1. \tag{5}$$

Here a and c are constants for a given system and they vary for different quark atoms. While index p remain same for all the systems. For example in the case of $c\bar{u}$, $c = 0.01675$ and $a = 0.01655$ while for the $b\bar{s}$, $c = 0.003488$ and $a = 0.003614$. It is very interesting to note that the index do not change from system to system, indicating it as characteristic property of the medium.

3. The dielectric function and confinement potential : A correspondence

Here an attempt has been made to achieve a correspondence between the quark confinement potential with the colour dielectric function discussed in the previous section. The quark potential in a Dirac equation with Lorentz scalar plus the vector character has been expressed in a bispinor form as [8–10],

$$(\sigma.p)\chi = (E+M)\phi\,,$$
$$(\sigma.p)\phi = (E-M-V(r))\chi\,, \tag{6}$$

where (χ, ϕ) represents the Dirac bispinors and, $V(r)$ corresponds to the quark confinement potential. As the potential appears in the Dirac equation, the dielectric function of the colour field appears through the Maxwell's equations given by

$$\nabla \times \boldsymbol{E} = i\omega \boldsymbol{B} \quad \text{and} \quad \nabla = -\,i\omega\varepsilon(r)\,\boldsymbol{E}\,. \tag{7}$$

The curl operator here can be expressed in terms of the spin matrix of a vector field as $\boldsymbol{S}.\boldsymbol{P}$. Where $\boldsymbol{S}$ corresponds to the spin matrix for the vector field given by [11],

$$S_x = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & -i \\ 0 & i & 0 \end{pmatrix}, S_y = \begin{pmatrix} 0 & 0 & i \\ 0 & 0 & 0 \\ -i & 0 & 0 \end{pmatrix}, S_z = \begin{pmatrix} 0 & -i & 0 \\ i & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \tag{8}$$

and $\boldsymbol{p} = -\,i\boldsymbol{\nabla}$.

Accordingly, the two Maxwell's equations can be expressed in terms of the vector spin as

$$(\boldsymbol{S}.\boldsymbol{P})\boldsymbol{E} = i\,\omega\boldsymbol{B}$$

and

$$(\boldsymbol{S}.\boldsymbol{P})\boldsymbol{B} = -\,\omega\,\varepsilon(r)\,i\,\boldsymbol{E}\,. \tag{9}$$

Now comparing eqs. (6) and (9) it is possible to associate

$$\Phi \longrightarrow i\boldsymbol{B}\,,$$
$$\chi \longrightarrow \boldsymbol{E}\,,$$
$$\omega \longrightarrow E+M$$

and

$$\omega\varepsilon(r) \longrightarrow E-M-V(r)\,. \tag{10}$$

Hence, we associate

$$\varepsilon\,(r) \longrightarrow \frac{E-M}{E+M} \quad \frac{V(r)}{E+M} \tag{11}$$

Now comparing the form of the dielectric function expressed in eq. (4) with that of eq. (11) for the colour dielectric function, we get the corresponding form of the quark confinement potential as

$$V(r) \longrightarrow ar^{p} \quad \text{where } p = 0.1 \tag{12}$$

such that the dynamical equations satisfied by the colour fields and the quarks in the confinement potential becomes similar.

4. Conclusion

From the present study of mesons we are able to deduce the property of the colour dielectric function. A general analytical form for the dielectric function has been obtained as $\varepsilon(r) = c - a_n r_n^P$ with $P = 0.1$ This behaviour of the colour dielectric function (as shown in Figure 1) is as expected by quantum chromo dynamics [7, 12]. As $r \to 0$ then $\varepsilon(r) \to c$, where $c > 1$ corresponds to the QED regime representing the asymptotic freedom of QCD. As $r \to (c/a)^{1/P}$ then $\varepsilon(r) \to 0$, corresponds to a perfect dielectric medium. In this case, the colour dielectric field is pushed inside the region leading to colour confinement [7, 9]. Similar semiclassical analysis for the colour confinement has been proposed by Jena and Pradhan [13].

Thus the dielectric function obtained from the present study allows no ionisation of the light quark from the quark atoms unlike the case of the QED atoms (ionisation of hydrogen *etc.*). It is seen from the tables that the Coulombic binding energy of the quarks increases at higher energy levels unlike the case of ordinary atoms (QED atoms). It is the infrared slavery of QCD that prevents the ionisation of quark atoms. These properties of the quarks are in general assumed phenomenologically through confinement potentials and the form of the potential obtained in Section 3 of this paper corresponds to the power law potential generally used for the study of heavy quark systems [14]. For example, a unified study of S and P-wave mesons for the confining potential of the form ar^β with $\beta = 0.3, 1, 2$ has been reported in literature [15]

One can have positively and negatively charged quark-atoms also ($c\bar{d}$, $b\bar{c}$, *etc.*). The baryons containing two light quarks and a heavy quark correspond to the helium like systems and can be studied in the same framework. The study of these quark-atoms are not only important in the spectroscopic point of view but are important as they are expected to be formed at the early stages of the universe and it may be possible to estimate or trace the primordial quark-atoms in cosmic ray showers [16]. The whole physics of atomic processes then revisit in the study of these quark atoms in a QCD medium.

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